Three Global Exponential Convergence Results of the GPNN for Solving Generalized Linear Variational Inequalities

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Abstract. The general projection neural network (GPNN) is a versatile recurrent neural network model capable of solving a variety of optimization problems and variational inequalities. In a recent article [IEEE Trans. Neural Netw., 18(6), 1697-1708, 2007], the linear case of GPNN was studied extensively from the viewpoint of stability analysis, and it was utilized to solve the generalized linear variational inequality with various types of constraints. In the present paper we supplement three global exponential convergence results for the GPNN for solving these problems. The first one is different from those shown in the original article, and the other two are improved versions of two results in that article. The validity of the new results are demonstrated by numerical examples.

1 Introduction

The following problem is called the *generalized linear variational inequality (GLVI)*: find $x^* \in \Re^m$ such that $Nx^* + q \in X$ and

$$(Mx^* + p)^T (x - Nx^* - q) \geq 0 \quad \forall x \in X, \tag{1}$$

where $M, N \in \Re^{m\times m}$; $p, q \in \Re^m$; and X is a closed convex set in $\Re^m$. It has many scientific and engineering applications, e.g., linear programming and quadratic programming [1], extended linear programming [2] and extended linear-quadratic programming [2, 3]. If X is a box set, i.e.,

$$X = \{x \in \Re^m | \underline{x} \leq x \leq \overline{x}\} \tag{2}$$

where $\underline{x}$ and $\overline{x}$ are constants (without loss of generality, any component of $\underline{x}$ or $-\overline{x}$ can be $-\infty$), a neurodyamic approach was proposed in [4] and [5] from different viewpoints for solving it. Moreover, in [5], the neurodynamic system was given a name, *general projection neural network (GPNN)*. A general form of the system is as follows:

$$\frac{dx}{dt} = \lambda W\{-Nx + \mathcal{P}_X((N - \alpha M)x + q - \alpha p) - q\}, \tag{3}$$

F. Sun et al. (Eds.): ISNN 2008, Part I, LNCS 5263, pp. 309–318, 2008.

where $\lambda \in \Re$, $W \in \Re^{m\times m}$ and $\alpha \in \Re$ are positive constants, and $\mathcal{P}_X(x) = (\mathcal{P}_{X_1}(x_1), \cdots, \mathcal{P}_{X_m}(x_m))^T$ with

$$\mathcal{P}_{X_i}(x_i) = \begin{cases} \underline{x}_i, \, x_i < \underline{x}_i, \\ x_i, \, \underline{x}_i \leqslant x_i \leqslant \overline{x}_i, \\ \overline{x}_i, \, x_i > \overline{x}_i. \end{cases} \tag{4}$$

Recently, the stability of the above GPNN was studied extensively in [6]. Many global convergence and stability results were presented. In addition, when X in the GLVI (1) is not a box set, but a polyhedral set defined by inequalities and equalities, several specific GPNNs similar to (3) were formulated to solve the corresponding problems. Some particular stability results of those GPNNs were also discussed. In the present paper, we will give a few new stability results of the GPNNs, reflecting our up-to-date progress in studying this type of neural networks.

Throughout the paper, $\|x\|$ denotes the l_2 norm of a vector x, I denotes the identity matrix with an appropriate dimension, and X^* stands for the solution set of GLVI (1), which is assumed to be nonempty. In addition, it is assumed that there exists at least one finite point in X^*. Define an operator $\mathcal{D}^+ f(t) = \limsup_{h\to 0^+}(f(t+h) - f(t))/h$, where $f(t)$ is a function mapping from $\Re \to \Re$.

2 Main Results

2.1 Box Set Constraint

First, we give a new stability result of the GPNN (3) for solving the GLVI with box-type constraint as described in (2). A useful lemma is introduced first [5,4].

Lemma 1. *Consider $\mathcal{P}_X : \Re^m \to X$ defined in (4). For any $u, v \in \Re^m$, we have $\|\mathcal{P}_X(u) - \mathcal{P}_X(v)\| \leq \|u - v\|$.*

Theorem 1. *Let $N = \{n_{ij}\}$ and $D = N - \alpha M = \{d_{ij}\}$. If*

$$n_{ii} > \sum_{j=1, j\neq i}^{m} |n_{ij}| + \sum_{j=1}^{m} |d_{ij}|, \quad \forall i = 1, \cdots, m, \tag{5}$$

then the GPNN (3) with $W = I$ is globally exponentially stable.

Proof. From (5) there exists $\theta > 0$ such that

$$n_{ii} \geq \sum_{j=1, j\neq i}^{m} |n_{ij}| + \sum_{j=1}^{m} |d_{ij}| + \theta, \quad \forall i = 1, \cdots, m. \tag{6}$$

Let x^* be a finite point in X^*, $L(t_0) = \max_{1\leq i\leq m} |x_i(t_0) - x_i^*|$ and $z_i(t) = |x_i(t) - x_i^*| - L(t_0)e^{-\lambda\theta(t-t_0)}$. In the following we will show $z_i(t) \leq 0$ for any $i = 1, \cdots, m$ and all $t \geq t_0$ by contradiction. Actually, if this is not true, there must exists a sufficiently small $\epsilon > 0$, two time instants t_1 and t_2 satisfying $t_0 \le t_1 < t_2$, and at least one $k \in \{1, \cdots, m\}$, such that

$$z_k(t_1) = 0, \quad z_k(t_2) = \epsilon \tag{7}$$

$$\mathcal{D}^+ z_k(t_1) \ge 0, \quad \mathcal{D}^+ z_k(t_2) > 0 \tag{8}$$

$$z_i(s) \le \epsilon, \quad \forall i = 1, \cdots, m; \quad t_0 \le s \le t_2. \tag{9}$$

From (3) we have

$$dx/dt = \lambda\{-N(x - x^*) + \mathcal{P}_X(Dx + q - \alpha p) - \mathcal{P}_X(Dx^* + q - \alpha p)\} \tag{10}$$

and from Lemma 1 we have

$$\begin{aligned} &|\mathcal{P}_{X_i}(d_i x + q_i - \alpha p_i) - \mathcal{P}_{X_i}(d_i x^* + q_i - \alpha p_i)| \\ \le &|d_i(x - x^*)| \le \sum_{j=1}^{m} |d_{ij}||x_j - x_j^*|, \quad \forall i = 1, \cdots, m, \end{aligned} \tag{11}$$

where $d_i \in \Re^{1\times m}$ denotes the ith row of D. Without loss of generality, we assume $x_k(t_2) - x_k^* > 0$. (The case of $x_k(t_2) - x_k^* < 0$ can be reasoned similarly.) It follows from (7), (9), (10) and (11) that $x_k(t_2) - x_k^* = L(t_0)e^{-\lambda\theta(t-t_0)} + \epsilon$, $|x_i(t_2) - x_i^*| \le L(t_0)e^{-\lambda\theta(t-t_0)} + \epsilon, \forall i = 1, \cdots, m$, and

$$\begin{aligned} \mathcal{D}^+ z_k(t_2) =& \mathcal{D}^+|x_k(t_2) - x_k^*| + \lambda\theta L(t_0)e^{-\lambda\theta(t-t_0)} \\ \le& -\lambda n_{kk}(x_k(t_2) - x_k^*) + \lambda \sum_{j=1, j\ne k}^{m} |n_{kj}||x_j(t_2) - x_j^*| \\ &+ \lambda \sum_{j=1}^{m} |d_{kj}||x_j(t_2) - x_j^*| + \lambda\theta L(t_0)e^{-\lambda\theta(t-t_0)} \\ \le& -\lambda n_{kk}(L(t_0)e^{-\lambda\theta(t-t_0)} + \epsilon) + \lambda \sum_{j=1, j\ne k}^{m} |n_{kj}|(L(t_0)e^{-\lambda\theta(t-t_0)} + \epsilon) \\ &+ \lambda \sum_{j=1}^{m} |d_{kj}|(L(t_0)e^{-\lambda\theta(t-t_0)} + \epsilon) + \lambda\theta L(t_0)e^{-\lambda\theta(t-t_0)} \\ =& \lambda\left(-n_{kk} + \sum_{j=1, j\ne k}^{m} |n_{kj}| + \sum_{j=1}^{m} |d_{kj}| + \theta\right) L(t_0)e^{-\lambda\theta(t-t_0)} \\ &+ \lambda\left(-n_{kk} + \sum_{j=1, j\ne k}^{m} |n_{kj}| + \sum_{j=1}^{m} |d_{kj}|\right)\epsilon \end{aligned}$$

In view of (5) and (6), we have $\mathcal{D}^+ z_k(t_2) < 0$, which contradicts (8). Hence,

$$|x_i(t) - x_i^*| \le L(t_0)e^{-\lambda\theta(t-t_0)}, \quad \forall i = 1, \cdots, m; \quad t \ge t_0. \tag{12}$$

The proof is completed.

The above theorem is proved in the spirit of [7]. From the analysis it can be inferred that the convergence rate of (3) is at least $\lambda\theta$ where θ is the difference between the left and right hand sides of (5). Different from most of the results in [6], the exponential convergence rate here is expressed in terms of every component of the state vector separately, which provides a more detailed estimation than the results obtained by the usual Lyapunov method.

In the above proof, if we choose $L(t_0) = \|x(t) - x^*\|^2$, following similar arguments we can arrive at the following condition which assures the global exponential stability results as well: the minimum eigenvalue of $(N + N^T)/2$ is greater than $\|D\|$. Interestingly, this is a result stated in Corollary 1 of [6] where a different proof was given.

2.2 General Constraints

Consider the GLVI (1) with X defined as

$$X = \{x \in \Re^m | x \in \Omega_x, Ax \in \Omega_y, Bx = c\}, \tag{13}$$

where $A \in \Re^{h\times m}, B \in \Re^{r\times m}, c \in \Re^r$, and Ω_x, Ω_y are two box sets defined as $\{x \in \Re^m | \underline{x} \le x \le \overline{x}\}$ and $\{y \in \Re^h | \underline{y} \le y \le \overline{y}\}$, respectively (cf. (2)).

Let $\tilde{A} = (A^T, B^T)^T$ and

$$\tilde{M} = \begin{pmatrix} M & -\tilde{A}^T \\ 0 & I \end{pmatrix}, \tilde{p} = \begin{pmatrix} p \\ 0 \end{pmatrix}, \tilde{N} = \begin{pmatrix} N & 0 \\ \tilde{A}N & 0 \end{pmatrix}, \tilde{q} = \begin{pmatrix} q \\ \tilde{A}q \end{pmatrix},$$
$$\tilde{\Omega}_y = \{y \in \Re^{h+r} | (\underline{y}^T, c^T)^T \le y \le (\overline{y}^T, c^T)^T\}, \tilde{U} = \Omega_x \times \tilde{\Omega}_y.$$

It was shown in [6] that the GLVI can be converted to another GLVI with a box set $\tilde{U}$ only, and as a result, can be solved by using the following specific GPNN:

$$\frac{du}{dt} = \lambda W\{-\tilde{N}u + \mathcal{P}_{\tilde{U}}((\tilde{N} - \alpha\tilde{M})u + \tilde{q} - \alpha\tilde{p}) - \tilde{q}\}, \tag{14}$$

where $\lambda > 0, \alpha > 0, W \in \Re^{(m+h+r)\times(m+h+r)}$ are constants, $u = (x^T, y^T)^T$ is the state vector, and $\mathcal{P}_{\tilde{U}}(\cdot)$ is the activation function defined similarly as in (4). The output of the neural network is simply $x(t)$, the first part of the state $u(t)$.

In [6], it was proved that when $W = (\tilde{N} + \alpha\tilde{M})^T$, if $M^T N > 0$ then the output trajectory $x(t)$ of the neural network is globally convergent to the unique solution x^* of the problem (1). In the following, we show that if this condition holds, the convergence rate can be exponential by choosing an appropriate scaling factor λ. The proof is inspired by [8].

Theorem 2. *Consider GPNN* (14) *with* $W = (\tilde{N} + \alpha\tilde{M})^T$ *for solving the GLVI with* X *defined in* (13). *If* $M^T N > 0$ *and* λ *is large enough, then the output trajectory* $x(t)$ *of the neural network is globally exponentially convergent to the unique solution of the problem.*

Proof. It was shown in [6, Theorem 5] that the solution of the GLVI is unique, which corresponds to the first part of any equilibrium point of (14). Consider the function $V(u(t)) = \|u(t) - u^*\|^2/2$ where u^* is a finite equilibrium point of (14). Following a similar analysis procedure to that of Corollary 4 in [5] we can derive

$$\frac{dV(u(t))}{dt} \leq \lambda\{-\alpha(u-u^*)^T\tilde{M}^T\tilde{N}(u-u^*)-\|\mathcal{P}_{\tilde{U}}((\tilde{N}-\alpha\tilde{M})u+\tilde{q}-\alpha\tilde{p})-\tilde{N}u-\tilde{q}\|^2\}.$$

It follows that

$$\begin{aligned}\frac{dV(u(t))}{dt} &\leq \lambda\alpha\{-(u-u^*)^T\tilde{M}^T\tilde{N}(u-u^*)\} = \lambda\alpha\{-(x-x^*)^TM^TN(x-x^*)\}\\ &\leq \lambda\alpha\{-\beta\|x-x^*\|^2\},\end{aligned}$$

where $\beta > 0$ denotes the minimum eigenvalue of $(M^TN + N^TM)/2$. Then

$$V(u(t)) \leq V(u(t_0)) - \lambda\alpha\beta\int_{t_0}^{t}\|x(s)-x^*\|^2ds$$

and

$$\|x(t)-x^*\|^2 \leq 2V(u(t_0)) - 2\lambda\alpha\beta\int_{t_0}^{t}\|x(s)-x^*\|^2ds.$$

Without loss of generality it is assumed $\|x(t_0) - x^*\|^2 > 0$ which implies $V(u(t_0)) > 0$. Then there exist $\tau > 0$ and $\mu > 0$ that depend on $x(t_0)$ only, so that $\int_{t_0}^{t_0+\tau}\|x(s) - x^*\|^2ds \geq \tau\mu$. If λ is large enough so that $\lambda \geq V(u(t_0))/(\alpha\beta\tau\mu)$, we have

$$V(u(t_0)) - \lambda\alpha\beta\int_{t_0}^{t_0+\tau}\|x(s)-x^*\|^2ds \leq 0.$$

It follows that for any $t > t_1 \geq t_0 + \tau$

$$\begin{aligned}\|x(t)-x^*\|^2 \leq&\|x(t_1)-x^*\|^2 + 2V(u(t_0)) - 2\lambda\alpha\beta\int_{t_0}^{t_1}\|x(s)-x^*\|^2ds\\ &- 2\lambda\alpha\beta\int_{t_1}^{t}\|x(s)-x^*\|^2ds\\ \leq&\|x(t_1)-x^*\|^2 + 2V(u(t_0)) - 2\lambda\alpha\beta\int_{t_0}^{t_0+\tau}\|x(s)-x^*\|^2ds\\ &- 2\lambda\alpha\beta\int_{t_1}^{t}\|x(s)-x^*\|^2ds\\ \leq&\|x(t_1)-x^*\|^2 - 2\lambda\alpha\beta\int_{t_1}^{t}\|x(s)-x^*\|^2ds.\end{aligned}$$

As a result,

$$\frac{\|x(t)-x^*\|^2 - \|x(t_1)-x^*\|^2}{t-t_1} \leq -2\lambda\alpha\beta\frac{f(t)-f(t_1)}{t-t_1}$$

where $f(t) = \int_{t_1}^{t} \|x(s) - x^*\|^2 ds$. Let $t \to t_1 + 0$, then we have

$$\frac{d\|x(t) - x^*\|^2}{dt} \leq -2\lambda\alpha\beta\|x(t) - x^*\|^2.$$

Therefore

$$\|x(t) - x^*\| \leq \|x(t_1) - x^*\|e^{-\lambda\alpha\beta(t-t_1)} = c_0 e^{-\lambda\alpha\beta(t-t_0)}, \quad \forall t > t_1$$

where $c_0 = \|x(t_1) - x^*\|e^{\lambda\alpha\beta(t_1-t_0)}$.

Since $dV(u(t))/dt \leq 0$, $u(t) \in \mathcal{S} = \{u \in \Re^m | V(u(t)) \leq V(u(t_0))\}$ for all $t \geq t_0$. Moreover, $V(u(t))$ is radially unbounded, then $\mathcal{S}$ is bounded, which implies that $\|x(t) - x^*\|$ is bounded over $t \geq t_0$. Let $\Delta = \max_{t_0 \leq t \leq t_1} \|x(t) - x^*\|$ and $c_1 = \Delta / e^{-\lambda\alpha\beta(t_1-t_0)}$. We have

$$\|x(t) - x^*\| \leq \Delta = c_1 e^{-\lambda\alpha\beta(t_1-t_0)} \leq c_1 e^{-\lambda\alpha\beta(t-t_0)}, \quad \forall t_0 \leq t \leq t_1.$$

Hence

$$\|x(t) - x^*\| \leq c_m e^{-\lambda\alpha\beta(t-t_0)}, \quad \forall t \geq t_0,$$

where $c_m = \max(c_0, c_1)$. The proof is completed.

2.3 Inequality Constraints

Consider X in (13) with inequality constraints only; i.e.,

$$X = \{x \in \Re^m | Ax \in \Omega_y\}, \tag{15}$$

where the notations are the same as in (13). Let

$$\hat{N} = ANM^{-1}A^T, \hat{q} = -ANM^{-1}p + Aq.$$

The following specific GPNN is proposed to solve the problem:

– State equation

$$\frac{du}{dt} = \lambda W\{-\hat{N}u + \mathcal{P}_{\Omega_y}((\hat{N} - \alpha I)u + \hat{q}) - \hat{q}\}; \tag{16a}$$

– Output equation

$$v = M^{-1}A^T u - M^{-1}p, \tag{16b}$$

where $\lambda \in \Re, \alpha \in \Re, \lambda > 0, \alpha > 0$ and $W \in \Re^{h \times h}$.

In [6], it was proved that when $W = (\hat{N} + \alpha I)^T$, if $M^T N > 0$ then the output trajectory $v(t)$ of the neural network is globally convergent to the unique solution x^* of the problem (1). In the following, we show that if this condition holds, the convergence rate can be exponential by choosing an appropriate λ.

Theorem 3. *Consider GPNN* (16) *with* $W = (\hat{N} + \alpha I)^T$ *for solving the GLVI with* X *defined in* (15). *If* $M^T N > 0$ *and* λ *is large enough, then the output trajectory* $v(t)$ *of the neural network is globally exponentially convergent to the unique solution of the problem.*

Proof. From [6, Theorem 6], the solution of the GLVI is unique, which is identical to $v^* = M^{-1}A^T u^* - M^{-1}p$ where u^* is any equilibrium point of (16a). Define a function

$$V(u(t)) = \frac{1}{2}\|u(t) - u^*\|^2, \quad t \geqslant t_0.$$

From (16b), we have

$$\|v - v^*\|^2 = \|M^{-1}A^T(u - u^*)\|^2 \leqslant \|M^{-1}A^T\|^2\|u - u^*\|^2.$$

Thus $V(u) \geqslant \frac{\|v-v^*\|^2}{2\|M^{-1}A^T\|^2}$. Following a similar analysis to that of Corollary 4 in [5] we can deduce

$$\frac{dV(u(t))}{dt} \leqslant \lambda\{-\alpha(u - u^*)^T\hat{N}(u - u^*) - \|\mathcal{P}_{\Omega_y}((\hat{N} - \alpha I)u + \hat{q}) - \hat{N}u - \hat{q}\|^2\}.$$

It follows that

$$\begin{aligned}\frac{dV(u(t))}{dt} &\leqslant \lambda\alpha\{-(u - u^*)^T ANM^{-1}A^T(u - u^*)\}\\ &= \lambda\alpha\{-[M^{-1}A^T(u - u^*)]^T M^T N[M^{-1}A^T(x - x^*)]\}\\ &= \lambda\alpha\{-(v - v^*)^T M^T N(v - v^*)\} \leqslant \lambda\alpha\{-\beta\|v - v^*\|^2\},\end{aligned}$$

where $\beta > 0$ denotes the minimum eigenvalue of $(M^TN + N^TM)/2$. Then

$$V(u(t)) \leq V(u(t_0)) - \lambda\alpha\beta\int_{t_0}^{t}\|v(s) - v^*\|^2 ds$$

and

$$\|v(t) - v^*\|^2 \leq 2\gamma V(u(t_0)) - 2\lambda\alpha\beta\gamma\int_{t_0}^{t}\|v(s) - v^*\|^2 ds,$$

where $\gamma = \|M^{-1}A^T\|^2$. The rest of the proof is similar to the latter part of the analysis of Theorem 2, and is omitted for brevity.

3 Illustrative Examples

Example 1. Let's first solve a GLVI (1) with a box set, where

$$M = \begin{pmatrix} 4 & 2 & -1\\ 0 & 3 & 0\\ -1 & 3 & 6\end{pmatrix}, N = \begin{pmatrix} 5 & 2 & -1\\ 1 & 5 & 0\\ -1 & 3 & 8\end{pmatrix}, p = \begin{pmatrix} -1\\ 2\\ 5\end{pmatrix}, q = \begin{pmatrix} 0\\ 2\\ 0\end{pmatrix},$$

and $X = \{x \in \Re^3 | (-4, 0, -4)^T \leqslant x \leqslant (6, 6, 6)^T\}$. Let $\alpha = 1$, it is easy to verify that the condition in Theorem 2 is satisfied. Actually, $n_{11} - |n_{12}| - |n_{13}| - \sum_{j=1}^{3}|d_{1j}| = 1, n_{22} - |n_{21}| - |n_{23}| - \sum_{j=1}^{3}|d_{2j}| = 1, n_{33} - |n_{31}| - |n_{32}| - \sum_{j=1}^{3}|d_{3j}| = 2$. Then the GPNN (3) is globally exponentially stable. All numerical simulations validated

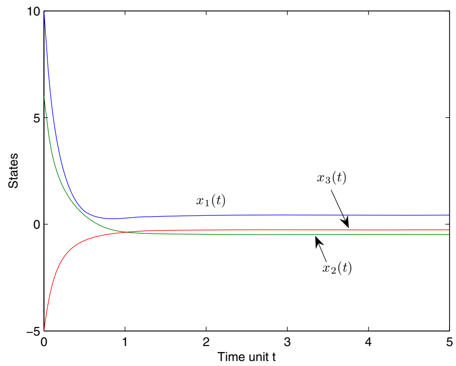

Fig. 1. State trajectories of the GPNN (3) in Example 1 with $W = I$, $\lambda = \alpha = 1$ and $x(0) = (10, 6, -5)^T$

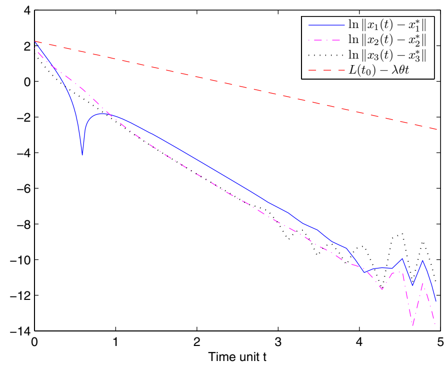

Fig. 2. Solution error of the GPNN (3) in Example 1. The estimated upper bound (*dashed line*) is also plotted.

this conclusion. Fig. 1 demonstrates the state trajectories started from the initial point $x(0) = (10, 6, -5)^T$ with $\lambda = 1$ (t_0 is set to 0), which converge to the unit solution of the problem $x^* = (0.4265, -0.4853, -0.2647)^T$. To show their exponential convergence rates, we take the natural logarithm of both sides of (12),

$$\ln |x_i(t) - x_i^*| \leq \ln L(t_0) - \lambda\theta t, \quad \forall i = 1, \cdots, 3; \quad t \geq 0.$$

and depict both sides of above inequality in Fig. 2. (It is evident that θ can be chosen as $\theta = 1$). The right-hand-side quantity now becomes a straight line in the figure. It is seen that the error of the states are all upper bounded by this line.

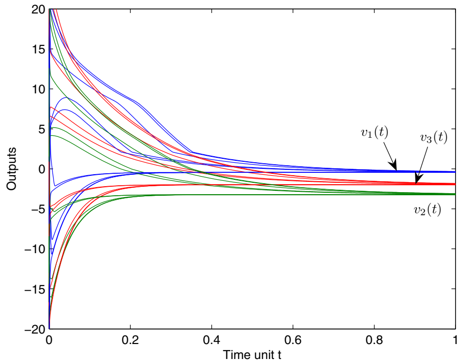

Fig. 3. Output trajectories of the GPNN (16) in Example 2 with $W = (\hat{N} + \alpha\hat{M})^T$, $\lambda = \alpha = 1$ and ten random initial points

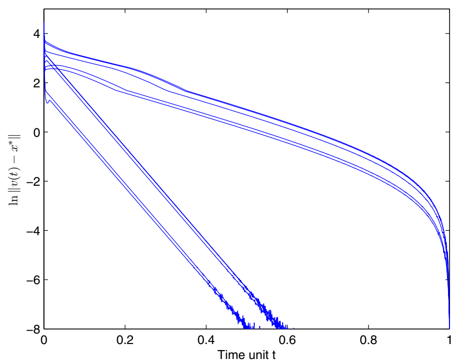

Fig. 4. Solution error of the GPNN (16) in Example 2. Because of numerical errors in simulations, when $\ln \|v(t) - x^*\| \leq -8$, the trajectories become unstable, and thus are not shown here.

Example 2. Consider a GLVI with a polyhedron set X defined in (15). Let

$$M = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}, N = \begin{pmatrix} 1 & -1 & -1 \\ 0 & -1 & 0 \\ 0 & 3 & -1 \end{pmatrix}, p = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}, q = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 1 & 1 & 0 \\ -5 & 5 & -1 \end{pmatrix},$$

and $\Omega_y = \{y \in \Re^2 | -10 \leqslant y \leqslant 10\}$. It can be verified that $M^T N > 0$. The GPNN (16) with $W = (\hat{N} + \alpha\hat{M})^T$ can be used to solve the problem according to Theorem 3. Simulation results showed that from any initial point this neural network globally converges to the unique equilibrium point $u^* = (-0.0074, -0.7556)^T$. Then, the solution of the GLVI is calculated as $x^* = (-0.4444, -3.2296, -1.9852)^T$. Fig. 3 displays the output trajectories of the neural network with $\lambda = \alpha = 1$ and 10 different initial points, and Fig. 4 displays the solution error (in natural logarithm) along with these trajectories. It is seen that for any of the 10 curves in Fig. 4 there exits a straight line with negative slope above it, that is, the convergence rate is upper bounded by an exponential function of t which tends to zero as $t \to \infty$.

4 Concluding Remarks

The general projection neural network (GPNN) has attracted much attention in recent years. The paper presents three sets of global exponential convergence conditions for it, which extend our recent results to some extent. Numerical examples illustrate the correctness of these new results.

Acknowledgments. The work was supported by the National Natural Science Foundation of China under the grant No. 60621062 and 60605003, the National Key Foundation R&D Projects under the grant No. 2003CB317007, 2004CB318108 and 2007CB311003, and the Basic Research Foundation of Tsinghua National Laboratory for Information Science and Technology (TNList).

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